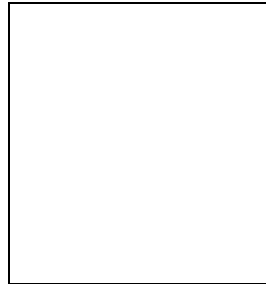


ENERGETICS OF NEUTRINOS IN NEUTRON STARS

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I review our proof that long range forces induced by the exchange of massless neutrino-antineutrino pairs do not affect the stability of neutron stars.

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1 Introduction

Consider two test neutrons at rest, separated by the distance $d \gg 1/M_Z$. If neutrinos are massless, they induce (see Fig. 1) an effective long range interaction between the two neutrons which, on dimensional grounds, must fall like

$$V_{(2)}(d) \sim \frac{G_F^2}{d^5}$$

($G_F \approx 10^{-5} \text{GeV}^{-2}$ is the Fermi coupling.)¹ This interaction is very weak: at short distances, $d \lesssim 10^{-8} \text{cm}$, it is dominated by the direct exchange of a Z boson; at large distances, $d \gtrsim 10^{-8} \text{cm}$, the gravitational interaction (!) between the two test neutrons is dominant. Similarly, in the presence

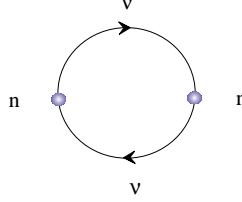


Figure 1: Exchange of a neutrino-antineutrino pair between two neutrons (n) at rest.

of N neutrons (taken to be at rest, to simplify), long range, multibody interactions are induced by the exchange of a neutrino-antineutrino pair between any subset of k neutrons, $k \leq N$.^b Consider then the contribution of these interactions to the total potential energy of the N neutrons. As each coupling to a neutron brings a factor of G_F , the k -body contribution gets smaller as k increases. However, each contributions has to be added with the adequate combinatoric factor, that, at least superficially, is proportional to the number of ways to take k neutrons out of N , $\propto N^k$ for large N . To be specific, consider a neutron star, that contains $N \approx 10^{57}$ neutrons – corresponding to about 1.4

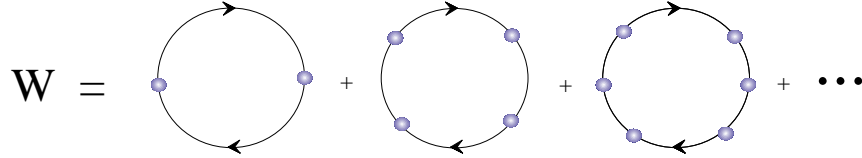


Figure 2: Expansion of the self-energy in k -body neutrino-antineutrino exchange.

solar mass– within a volume of radius $R \approx 10 \text{km}$, Fig.[3]. The self-energy of the neutron star from neutrino exchange is naively represented by the serie of Fig.[2], where the dots now mean insertions of the neutron density, ρ_N . The term with k insertions of the neutron density scales approximately as

$$W^{(k)} \sim \frac{C_k}{R} \left(\frac{G_F N}{R^2} \right)^k, \quad (1)$$

where C_k is a dimensionless numerical coefficient. The dimensionless parameter that governs the expansion is thus $G_F N / R^2 \sim G_F \rho_n R$. For a neutron star, this parameter is $\mathcal{O}(10^{12})$ (!).

Attempts to a direct summation of this series (truncated after N terms) have yielded enormous values for the interaction self-energy⁴, which led some authors to claim that neutrinos must have a mass of at least 0.4 eV in order to allow neutron stars to exist^{4,5,6,c}. Our approach to this problem

^bOne-loop diagrams with an odd number of neutron insertions vanish.

^cThis problem has an interesting story. It was first raised by R. Feynman in his Caltech lectures on gravity². He asked himself whether long range neutrino forces could mimic gravity, but ruled it out because the equivalence principle was not satisfied. He noticed that a large quantity of matter would render the expansion in k -body interactions divergent. Hartle addressed this problem in a cosmological context³. He noticed that the large scale repartition of matter in the Universe is such that the perturbative expansion is actually well-defined and leads to totally negligible effects. He also remarked that the long range neutrino interactions have no effect in presence of an homogeneous distribution of matter. Fischbach later recognized that the expansion parameter is $\gg 1$ inside of a neutron star⁴.

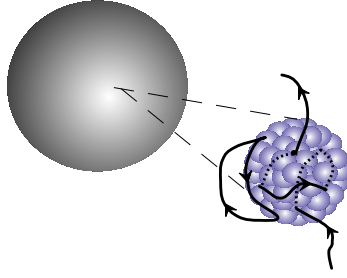


Figure 3: Exchange of neutrino-antineutrino pairs between the neutrons of a neutron star.

is different. Following Abada *et al*, we claim that the series represented in Fig.[2] is meaningless if $G_F \rho_N R \gtrsim 1$, and must be resummed in a non-perturbative way to get a sensible result^{8,7}. We have performed this resummation and find that the apparent infrared divergence is an artifact of the expansion and that the energy is finite and well-behaved. This implies in particular that there is no lower bound on the mass of the neutrinos. Along the way we have encountered some interesting physics. In particular, we show that the ground state of the system has a non-zero neutrino charge – a result which was previously anticipated by Loeb⁹. In our simple model for the density of the neutron star it is straightforward to calculate both the energy and the neutrino number of a neutron star. Let us mention that several groups have examined various aspects of this problem^{8,10,11,12,13,14}. In particular, Arafune and Mimura¹⁴ have confirmed our asymptotic result using an analytical approximation.

2 The neutrino ground state in a neutron star

We want to compute the contribution to the self-energy of the neutron star due to long range neutrino forces, or equivalently, the shift in the neutrino ground state energy in presence of a neutron star. The latter can be defined in terms of the neutrino Hamiltonian $H_{(0)}$ in the presence (absence) of the star¹⁵:

$$W = \langle \hat{0} | H | \hat{0} \rangle - \langle 0 | H_0 | 0 \rangle. \quad (2)$$

Here $|\hat{0}\rangle$ denotes the neutrino ground state in presence of the star, while $|0\rangle$ denotes the usual neutrino vacuum state. As we have already alluded, the state $|\hat{0}\rangle$ contains in general a non-zero neutrino number (i.e., it is “charged”). Note that the expression in Eq. (2) is a formal, ultraviolet (UV) divergent quantity which needs to be renormalized. This renormalization may be done using the usual techniques of quantum field theory.

In order to proceed, it is convenient to introduce a low energy effective Lagrangian for the neutrino field. After integrating out all of the other particles in the theory, one obtains^{16,8,17}

$$\mathcal{L}_{\text{eff}} = \bar{\psi}_L \left[i \not{\partial} + \alpha \gamma^0 \right] \psi_L \quad (3)$$

where $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$, and where

$$\alpha(\vec{x}) = G_F \rho_n(\vec{x}) / \sqrt{2} \sim 20 \text{ eV} \quad (4)$$

is the electroweak potential induced by the finite neutron density (typically $\rho_n \approx 0.4 \text{ fm}^{-3}$ in a neutron star). This potential is identical to the one which is usually considered in the well-known Mikheyev-Smirnov-Wolfenstein (MSW) effect¹⁸. The potential term in Eq.(3), which is attractive for neutrinos and repulsive for antineutrinos, resembles a position-dependent chemical potential, and it is maybe not surprising that the ground state of the system can carry a non-zero neutrino number.^{d e}

^dThis conclusion could be premature. For instance, in 1 + 1 dimension, such an external potential has absolutely no physical effect (if the neutrinos are massless)¹¹.

^eFrom the effective Lagrangian point of view, it is manifest that an homogeneous distribution of neutrons, $\rho_n = \text{const.}$ has no net effect: it can simply be gauged away. (See Hartle’s comment in footnote b.)

The Schwinger-Dyson expansion for W in terms of the potential $\alpha(\vec{x})$ and the neutrino propagator $G_0(\vec{x}, \vec{x}'; \omega)$ gives⁷

$$W = \frac{1}{2\pi i} \sum_{k=1}^{\infty} \frac{1}{k} \int_C d\omega \text{Tr}_{\mathbf{x}} [\alpha G_0(\omega)]^k \quad (5)$$

$$\equiv \sum_k W^{(k)}. \quad (6)$$

This series corresponds precisely to the one which is represented diagrammatically in Fig.[2]. A useful aspect of the perturbative expansion is that it isolates the UV divergence in W . In particular, the only UV divergent term in Eq.(6) is that with $k = 2$ which is related to the vacuum polarization of the Z boson in the complete theory. While the terms with $k \geq 4$ are separately UV finite, their sum is ill-defined for $\alpha R \gtrsim 1$. The non-perturbative resummation of these terms is the main goal of our calculation.

2.1 The 2-body contribution.

We begin by an estimate of the two-body contribution to the self-energy of a neutron star, W_2 . From Eq.(3) this contribution is UV divergent, $W_2 \sim \Lambda^2$, with Λ some UV cut-off. The procedure to compute this term is in principle well-defined. The diagram can be computed and renormalized both in the effective theory and in the underlying theory – *i.e.* the Standard Model – and the respective results must be matched at some scale. A natural scale for matching is the inverse size of the neutrons, $\Lambda \sim 1\text{GeV}$. From the effective Lagrangian point of view, this implies that the coarse grained structure of the star is taken into account. At larger distances, $d \gg \text{GeV}^{-1}$ it is however sufficient to consider a smooth, continuous distribution of neutrons. This simplification is helpful when we come to the evaluation of the higher order, UV finite, multibody contributions.

A rough estimate for W_2 is^{4,8,7,13}

$$W_2 \sim +\alpha^2 \Lambda^2 R^3 \quad (7)$$

The two-body interactions give a contribution $\propto R^3$, and proportional to the number of neutron pairs in the star, $\propto \rho^2 \sim N^2$. Also, because the two-body interaction is repulsive, this contribution is positive. For a neutron star, this estimate gives a negligible contribution,

$$W_2 \sim 10^{16} \text{kg} \ll 1.4 M_{\odot} \sim 10^{31} \text{kg}. \quad (8)$$

2.2 The k -body contributions, $k \geq 4$.

The terms in the Schwinger-Dyson expansion with $k \geq 4$ are UV finite. However, there are sensitive to the size of the star, $\propto (\alpha R)^k$. For $\alpha R > \mathcal{O}(1)$ these terms must be resummed in a non-perturbative way. We begin by making some useful simplifications. First, to study long distance, UV finite effects, we can forget the neutrons and only consider their mean field effect: $\alpha(x)$ is taken to be a smooth function of \mathbf{x} . Furthermore, we approximate the shape of the star by a spherical square well potential, with depth $\alpha \approx 20\text{eV}$ and radius R . For $k \geq 4$, the only dimensionless parameter is αR . If $\alpha R \lesssim 1$ the Schwinger-Dyson expansion is well-defined (the "weak coupling regime"), while it must be resummed if $\alpha R \gtrsim 1$ (the "strong coupling regime"). If we want to understand the transition from the "weak coupling" to the "strong coupling" regime, we only have consider systems of rather small size: *e.g.* for fixed $\alpha \approx 20\text{eV}$, $0.1 \leq \alpha R \leq 100$. The extrapolation to $R \sim 10\text{km}$ will be trivial.

To resum the $k \geq 4$ terms, we use the following expression –adapted from Schwinger¹⁵– for the shift of the neutrino vacuum energy in presence of the external electroweak potential:

$$W = \frac{1}{2\pi} \sum_{l=0}^{\infty} (2l+2) \int_0^{\infty} d\omega [\delta_l(\omega) + \delta_l(-\omega)]. \quad (9)$$

where $\delta_l(\pm\omega)$ is the scattering phase shift of an incident neutrino (antineutrino) of energy $(-)\omega$ and l labels the orbital angular momentum. The factor $(2l+2) \equiv (2j+1)$ is the degeneracy factor for a given energy $\pm\omega$ and total angular momentum j .

The details of our calculations can be found in⁷. Here, I only summarize the main results.

- Eq. (9) is a formal UV -divergent expression which need to be renormalized. We achieved this by subtracting the leading $W_2 = \mathcal{O}(\alpha^2)$ term in the Born expansion of the phase shifts. The "renormalized" term W_2 is then like in Eq.(7).
- If the neutrino are massless, there are no bound states, only resonances. This is because a neutrino must be able to flip chirality $\nu_L \rightarrow \nu_R$ in order to form a bound state. Resonances exists because no chirality flip is necessary to bend the trajectory of a massless neutrino. (See Loeb⁹.) These resonances become extremely narrow – *i.e.* long lived – as R increases.
- The physical implication of these resonances is that a non-zero neutrino charge can be "confined" within the potential/neutron star. Intuitively, the external electroweak potential induced by the finite neutron density polarizes the neutrino vacuum. If the external field is strong enough, *i.e.* $\alpha R \gtrsim 1$, the neutrino vacuum is unstable toward the creation of neutrino-antineutrino pairs. In an open system, like a neutron star, the antineutrinos fly away to spatial infinity while the neutrinos effectively "screen" the external electroweak potential. This effect is obviously non-perturbative in the external potential.

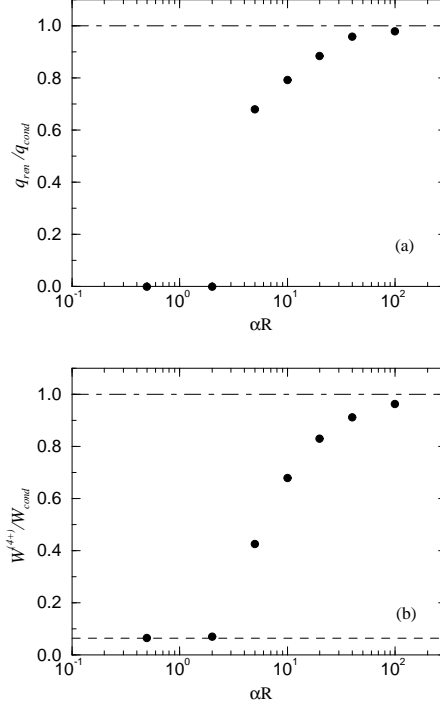


Figure 4: Renormalized exact charge, Q/Q_ν , and self-energy $|(W - W_2)/W_\nu|$ as a function of αR .

- As R increases, we observe a first order phase transition at $\alpha R \sim 1$ between the "weak coupling" and the "strong coupling" regimes, after which the self-energy rapidly reaches an asymptotic behaviour, Fig.[4]. The calculations show that this asymptotic value coincides with the thermodynamic limit of a ideal Fermi gas of massless neutrinos with chemical potential α , trapped within a volume $V = 4/3\pi R^3$:

$$\begin{aligned}
 W - W_2 &\approx W_\nu = -\frac{\alpha^4 R^3}{18\pi} \\
 Q &\approx Q_\nu = 2\frac{(\alpha R)^3}{9\pi}
 \end{aligned} \tag{10}$$

This behaviour for very large αR was anticipated by Abada *et al*⁸ and confirmed by Arafune and Mimura¹⁴.

- Because there is no other dimensionless parameter in the theory than αR , it is trivial to extrapolate these results to a realistic neutron star, $R \sim 10km$. There are about

$$Q_\nu \approx 10^{36} \quad (11)$$

low energy, $\omega \sim eV$, neutrinos trapped in a neutron star. Expressed in kilograms, they contribute about

$$W - W_2 \approx -30kg \quad (12)$$

(for each generation of massless neutrinos) to the mass of the star, to be compared to a total mass of $10^{31}kg$.

- This effect is extremely small, probably of no observable consequence. Incidentally, this proves that massless neutrinos can not affect the gravitational stability of a neutron star.

3 Conclusions

The physics underlying the resummation problem is non-perturbative. The neutrons are the source of a neutral electroweak effective potential that polarizes the neutrino vacuum. When the system is large/dense enough (the "strong coupling" regime), like in a neutron star, the energy of the system is lowered by spontaneous creation of neutrino-antineutrino pairs. In an open system, again like a neutron star, the ground state contains a "neutrino condensate". These results show that there is no "mysterious" long range neutrino force at work in a neutron star¹⁹.

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